

# Comparing general equilibrium modeling in Julia and GAMS: An example using CSAVE

Y.-H. Henry Chen

Research Scientist, MIT CS3

January 23, 2026

# Message from GAMS...

*“The reason for GAMS superior performance in [having the shorter model generation time of] this example [IJKLM model] is the use of relational algebra... to process complex queries efficiently.”*

— Broihan (2023), GAMS Corporation

# Response from Julia...

*“... it is not difficult to address it [the bottle neck of Broihan’s benchmark Julia model], given that general-purpose languages like Julia and Python have libraries specialized for this task.”*

— M. Lubin, O. Dowson, J. D. Garcia, J. Huchette, B. Legat (2023), JuMP developers

# Questions

- What does a CGE written in Julia look like?
- Which language is faster in running CSAVE?
- How coding strategies may affect Julia's solving time?

# Scope

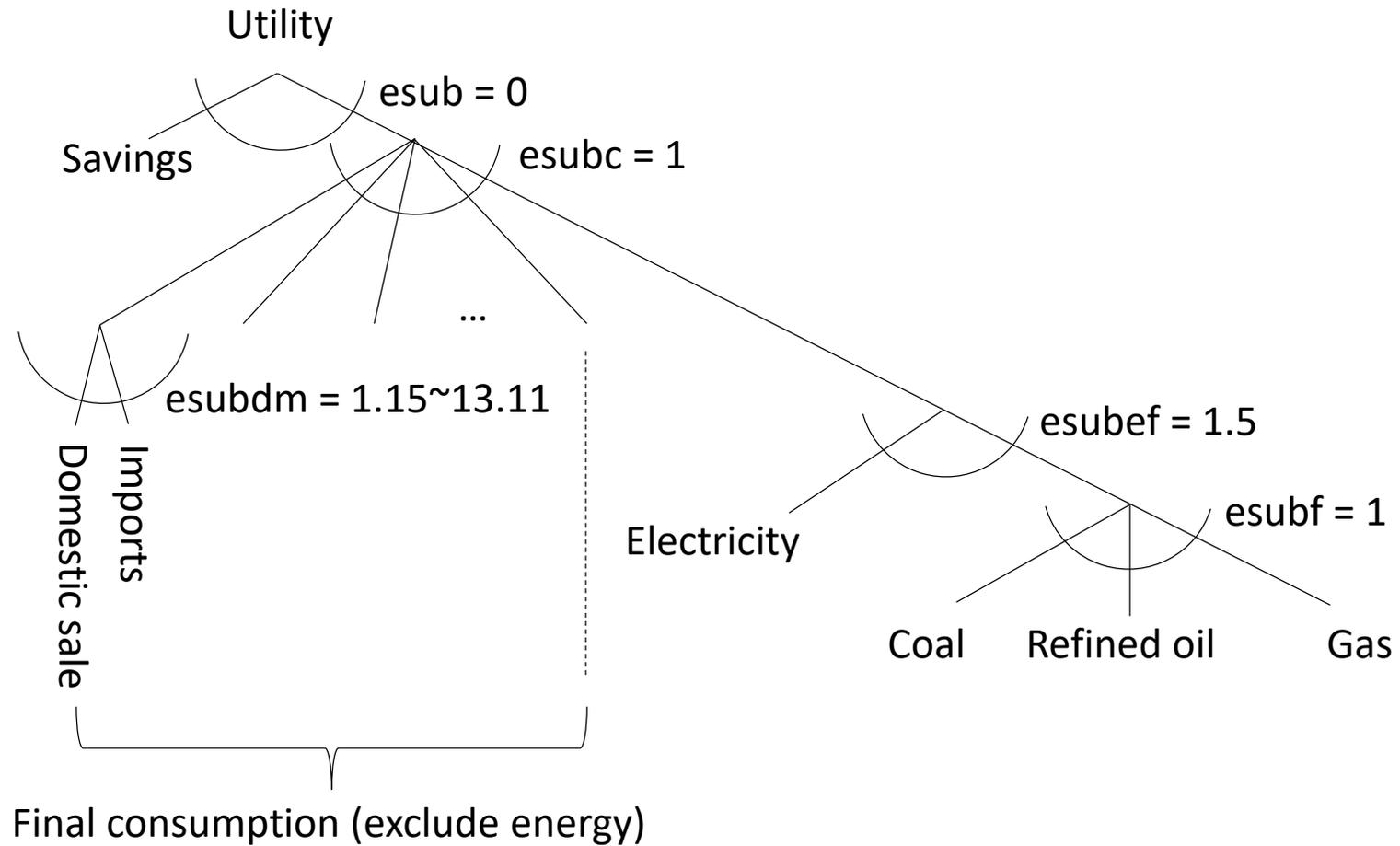
- An exercise based on CSAVE (Chen and Paltsev, 2025): in Julia vs. in GAMS
- Model generation & solving time may vary across models & computers
- Technical details/coding tricks are left for a modeling workshop

# Model

- CSAVE: Chen & Paltsev (2025)
  - Motivated by EPPA
  - Derived from GTAPinGAMS
  - In this exercise: multi-sector & multi-region — ↑ resolution for “stress testing”
  - GTAP9 database

# Model

Expenditure function





# Julia vs. GAMS

- Julia

- Developed by J. Bezanson, S. Karpinski, V. B. Shah, A. Edelman in 2012
- Open source/free
- General purpose w/ packages for extension
- MPSGE.jl is a package of Julia
- Free PATH license until 12/31/2026 for now

- MPSGE.jl

- D. Anthoff, E. Lazarus, M. Phillipson

- GAMS

- Developed by Alex Meeraus at the World Bank in the 1970s
- Proprietary
- DSML for optimization problems
- MPSGE is a sub-system of GAMS
- Access PATH via GAMS license

- MPSGE

- Tom Rutherford

# Coding in Julia

- Packages used directly include

- I/O & data processing

- CSV
- Dataframes
- JLD2
- *CSVtoDIC*
- *GTAP9data*

- Model construction and solving

- JuMP
- MPSGE
- PATHSolver

- Generating figures

- StatsBase
- StatsPlots
- Plots
- PyPlot
- Measures
- Distributions

- Time measurement

- BenchmarkTools

# Coding in Julia

- CSAVEinJulia is packaged and defines its own environment
  - Project.toml — names & identities of the direct dependencies
  - Manifest.toml — the dependency graph, dependency version, where to load

## Project.toml

```
[deps]
CSV = "336ed68f-0bac-5ca0-87d4-7b16caf5d00b"
DataFrames = "a93c6f00-e57d-5684-b7b6-d8193f3e46c0"
GTAPdata = "130c15d4-ad42-4bd4-91d3-6787eb393e58"
Ipopt = "b6b21f68-93f8-5de0-b562-5493be1d77c9"
JLD2 = "033835bb-8acc-5ee8-8aae-3f567f8a3819"
JuMP = "4076af6c-e467-56ae-b986-b466b2749572"
MPSGE = "d5dc2f44-7ae2-49e9-bc77-b47b6bca565d"
```

## Manifest.toml

```
[[deps.GTAPdata]]
deps = ["CSV", "CSVtoDIC", "DataFrames", "JLD2"]
git-tree-sha1 = "d5cb8e98efe4a2d2de8cfabcb153f6ba5ac84558"
repo-rev = "master"
repo-url = "https://github.com/chenyhmittedu/GTAPdata"
uuid = "130c15d4-ad42-4bd4-91d3-6787eb393e58"
version = "0.1.0"

[[deps.HashArrayMappedTrie]]
```

Pin package "GTAPdata" to a specific commit



CSAVEinJulia is available on <https://github.com/chenyhmittedu/>

# Coding in Julia

GAMS' set operation is much simpler and cleaner:

```
vtw(j)      = sum(r, vst(j,r));  
  
vafm(i,g,r) = vdfm(i,g,r)*(1+rtfd(i,g,r))+vifm(i,g,r)*(1+rtfi(i,g,r));
```

Julia uses dictionary (key-value mapping) to accomplish the same task:

```
vtw      = Dict{  
    j => sum(vst[(j, r)] for r ∈ set_r)  
    for j ∈ set_i  
    }  
  
vafm     = Dict{  
    (i, g, r) => vdfm[(i, g, r)]*(1+rtfd0[(i, g, r)])+vifm[(i, g, r)]*(1+rtfi0[(i, g, r)])  
    for i ∈ set_i, g ∈ set_g, r ∈ set_r  
    }
```

# Coding in Julia

## GAMS/MPSGE

```
$sectors:
y(g,r) $vom(g,r)           ! Supply
m(i,r) $vim(i,r)          ! Imports
yt(j) $vtw(j)             ! Transportation services
E(i,s,r) $vxmd(i,s,r)     ! Exports
A(i,g,r) $vafm(i,g,r)     ! Armington good i used k

$commodities:
p(g,r) $vom(g,r)          ! Domestic output price
pm(j,r) $vim(j,r)         ! Import price
pt(j) $vtw(j)             ! Transportation services
pf(f,r) $(evom(f,r) $mf(f)) ! Primary factors rent
ps(f,g,r) $(sf(f) and vfm(f,g,r)) ! Sector-specific primary
PX(i,s,r) $vxmd(i,s,r)    ! Price index for exports
PA(i,g,r) $vafm(i,g,r)    ! Price for Armington good
PE(g,r) $vxm(g,r)         ! Price index for exports

$consumers:
ra(r)                     ! Representative agent
```

## Julia/MPSGE.jl

```
@sectors(MGE, begin
    Y[set_g, set_r],      (description = "Supply")
    M[set_i, set_r],      (description = "Imports")
    YT[set_i],           (description = "Transportation services")
    E[set_i, set_r, set_r], (description = "Subsidy and transport service")
    A[set_i, set_g, set_r], (description = "Armington good")
end)

@commodities(MGE, begin
    P[set_g, set_r],      (description = "Domestic output price")
    PM[set_i, set_r],     (description = "Import price")
    PT[set_i],           (description = "Transportation services")
    PF[set_mf, set_r],    (description = "Non-sector-specific primary f")
    PS[set_sf, set_g, set_r], (description = "Sector-specific primary facto")
    PX[set_i, set_r, set_r], (description = "Price index for exports (incl")
    PA[set_i, set_g, set_r], (description = "Price index for Armington goo")
    PE[set_i, set_r],     (description = "Price index for exports (excl")
end)

@consumers(MGE, begin
    RA[set_r],           (description = "Representative agent")
end)
```

# Coding in Julia

## GAMS/MPSGE

```
$prod:y(g,r)$vom(g,r)  t:etadx(g)  s:esub(g)  sn(s):esubn(g)  sve(sn):esubve(g)  sva(sve):esubva(g)  sef(sve):esubef(g)  sf(sef):esubf(g)
o:P(g,r)                q:(vom(g,r)-vxm(g,r))                a:RA(r)  t:rto(g,r)
o:PE(g,r)                q:vxm(g,r)                            a:RA(r)  t:rto(g,r)
i:PA(i,g,r)$fe(i)        q:vafm(i,g,r)
i:PA(i,g,r)$elec(i)      q:vafm(i,g,r)                        sf:
i:PA(i,g,r)$ne(i)        q:vafm(i,g,r)                        sef:
i:ps(sf,g,r)             q:vfm(sf,g,r)  p:(1+rtf0(sf,g,r))                sn:
i:pf(mf,r)               q:vfm(mf,g,r)  p:(1+rtf0(mf,g,r))                sva:
```

## Julia/MPSGE.jl

```
for g ∈ set_i, r ∈ set_r
  @production(MGE, Y[g, r], [t = etadx[g], s = esub[g], sn => s = esubn[g], sve => sn = esubve[g], sva => sve = esubva[g], sef => sve = esubef[g], sf =
    @output(P[g, r],          vhm[g, r], t, taxes = [Tax(RA[r], rto[g, r])], reference_price = 1-rto0[g, r])
    @output(PE[g, r],        vxm[g, r], t, taxes = [Tax(RA[r], rto[g, r])], reference_price = 1-rto0[g, r])
    [@input(PA[i, g, r],     vafm[i, g, r], sf) for i ∈ set_fe]...
    [@input(PA[i, g, r],     vafm[i, g, r], sef) for i ∈ set_elec]...
    [@input(PA[i, g, r],     vafm[i, g, r], sn) for i ∈ set_ne]...
    [@input(PS[sf, g, r],    vfm[sf, g, r], s, taxes = [Tax(RA[r], rtf[sf, g, r])], reference_price = 1 + rtf0[sf, g, r]) for sf ∈ set_sf]...
    [@input(PF[mf, r],       vfm[mf, g, r], sva, taxes = [Tax(RA[r], rtf[mf, g, r])], reference_price = 1 + rtf0[mf, g, r]) for mf ∈ set_mf]...
  end
end
```

# Coding in Julia

## GAMS/MPSGE

```
$demand:ra(r)
    d:p("c",r)
    e:p("c","USA")
    e:p("g",r)
    e:p("i",r)
    e:ps(sf,j,r)
    e:pf(mf,r)
    q:vom("c",r)
    q:vb(r)
    q:(-vom("g",r))
    q:(-vom("i",r))
    q:vf(mf,j,r)
    q:evom(mf,r)
```

## Julia/MPSGE.jl

```
for r ∈ set_r
    @demand(MGE, RA[r], begin
        @final_demand(P[:c, r], vom[:c, r])
        @endowment(P[:c, :USA], vb[r])
        @endowment(P[:g, r], -vom[:g, r])
        @endowment(P[:i, r], -vom[:i, r])
        [@endowment(PF[f, r], evom[f, r]) for f ∈ set_mf]...
        [@endowment(PS[f, j, r], vfm[f, j, r]) for f ∈ set_sf, j ∈ set_i]...
    end)
end
```

# Setting

- Data resolution

- 56x2 — 2-region *USA, ROW*
- 56x4 — 4-region *USA, EUR, CHN, ROW*
- 56x8 — 8-region *USA, EUR, ROE, JPN, IND, CHN, ANZ, ROW*
- 56x16 — 16-region *USA, ..., ANZ, TWN, CAN, MEX, BRA, RUS, KOR, IDZ, ASI, ROW*
- 56x32 — 32-region *USA, [disaggregated EUR: DEU, FRA, GBR, ITA, ESP, ...], ..., ROW*
- 56x50 — 50-region *Disaggregate 56x32*
- 56x61 — 61-region *Disaggregate 56x50*
- 56x74 — 74-region *Disaggregate 56x61*

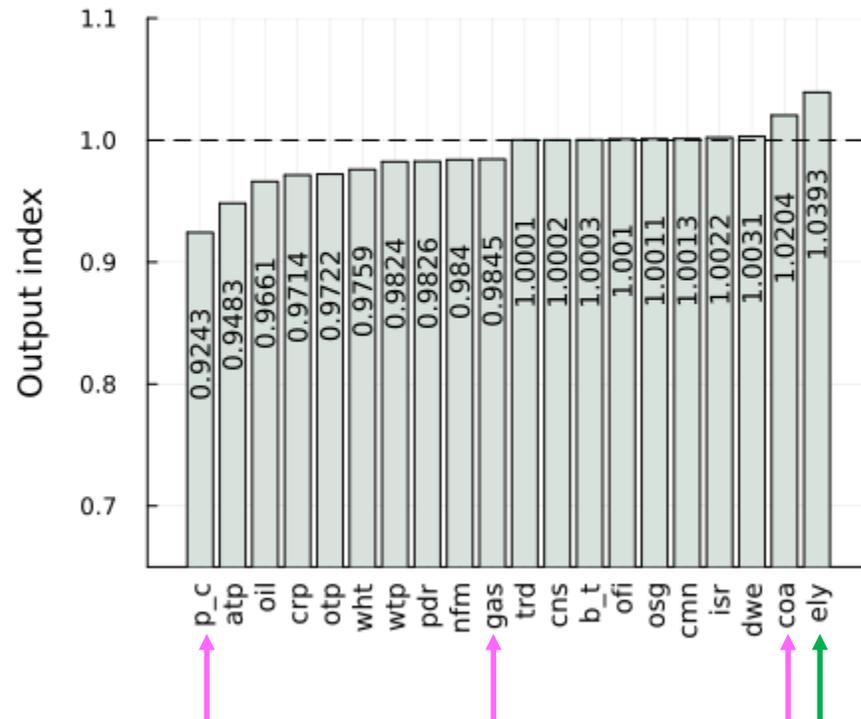
# Setting

- Scenario
  - US doubles tax rates on fossil fuels use:  
 $rtfd0[i, g, r] \times 2.0$  &  $rtfi0[i, g, r] \times 2.0$  ;  $r \in \text{USA}$ ,  $i \in \text{coa}$ ,  $p\_c$ ,  $\text{gas}$

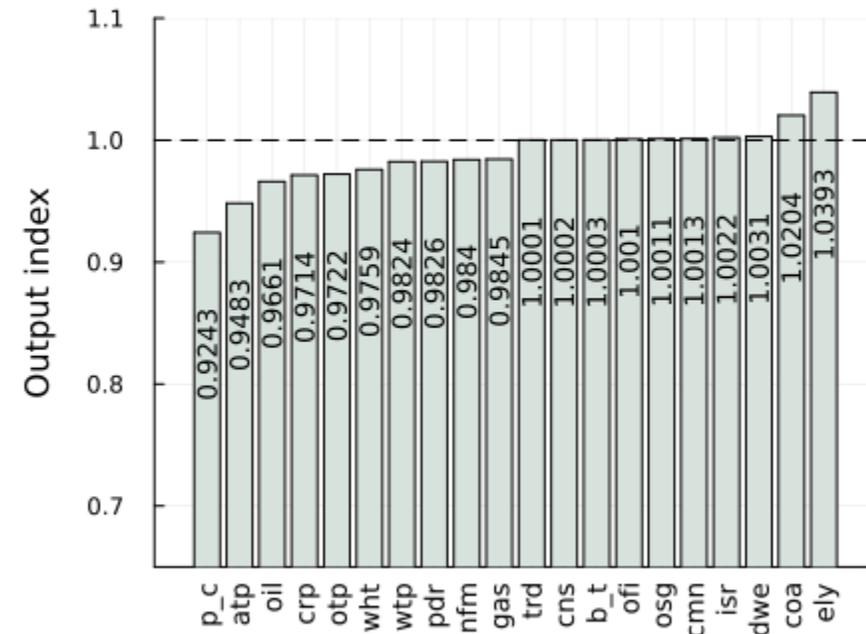
# Simulation

## Double fossil fuel tax rates

Selected sectoral outputs of USA:  
CSAVEinJulia



Selected sectoral outputs of USA:  
CSAVEinGAMS



# Computer spec

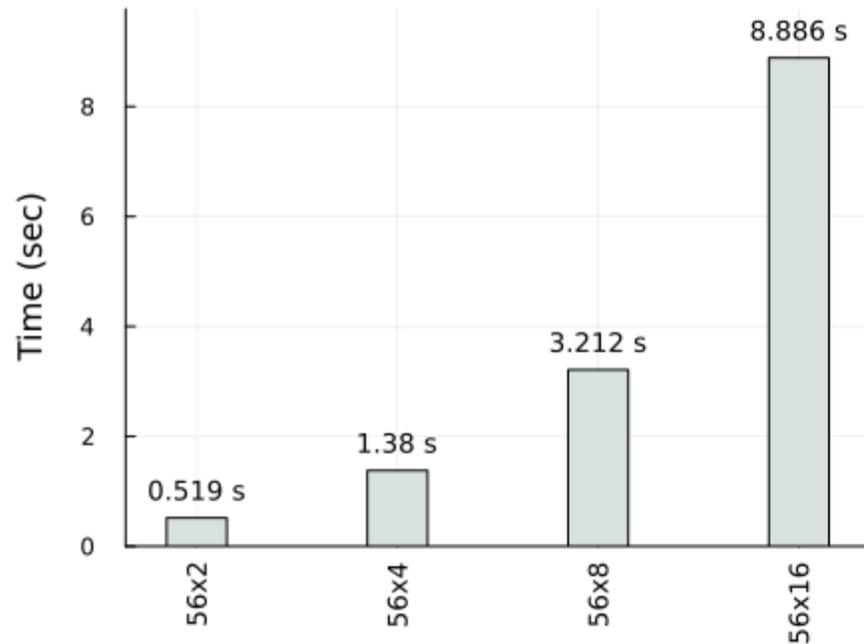
- CPU : AMD Ryzen Threadripper PRO 7955WX 16-Cores 4.5 GHz
- RAM : 128 GB 4800 MT/s



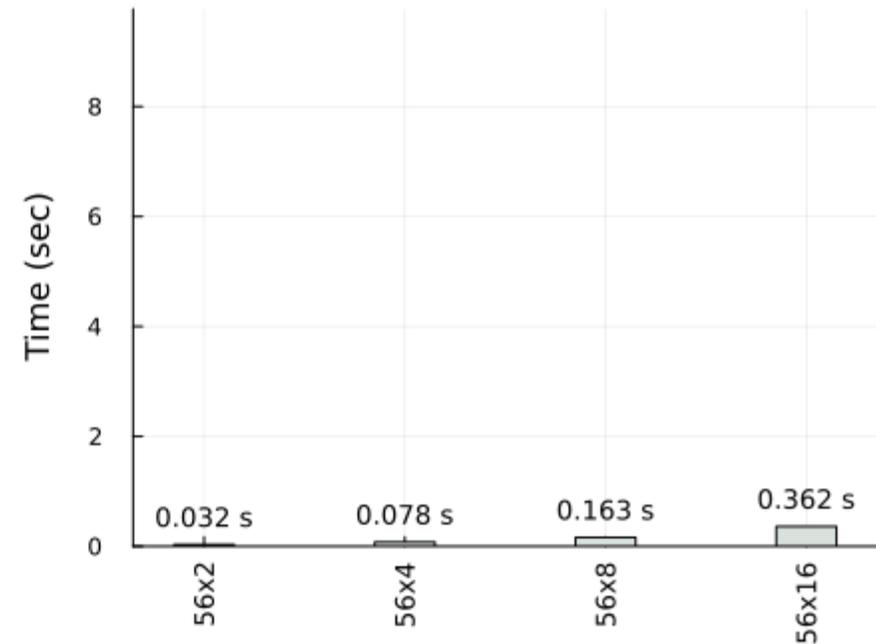
# Model generation time

For now, each data resolution setting is run once for demonstration purposes.

Model generation time:  
CSAVEinJulia

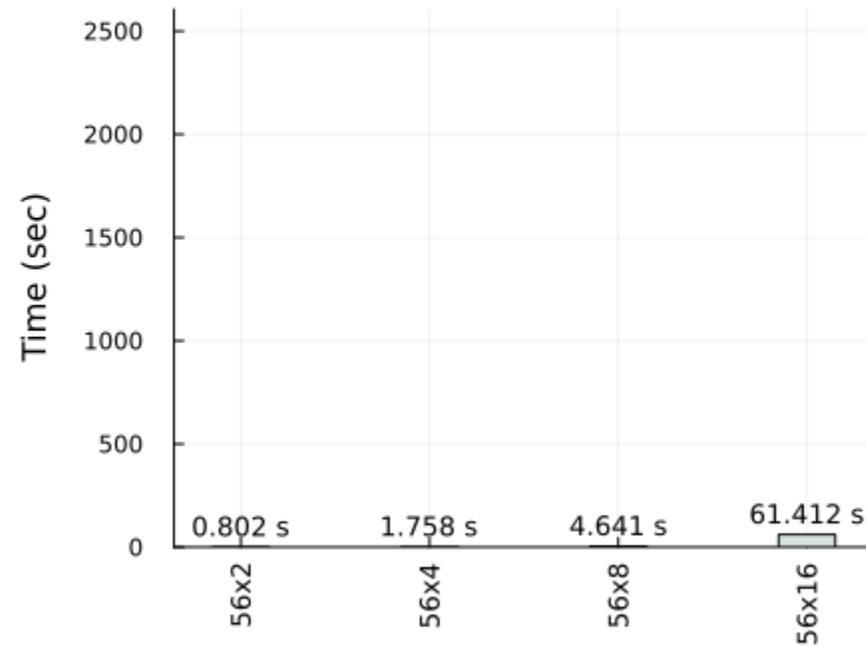


Model generation time:  
CSAVEinGAMS

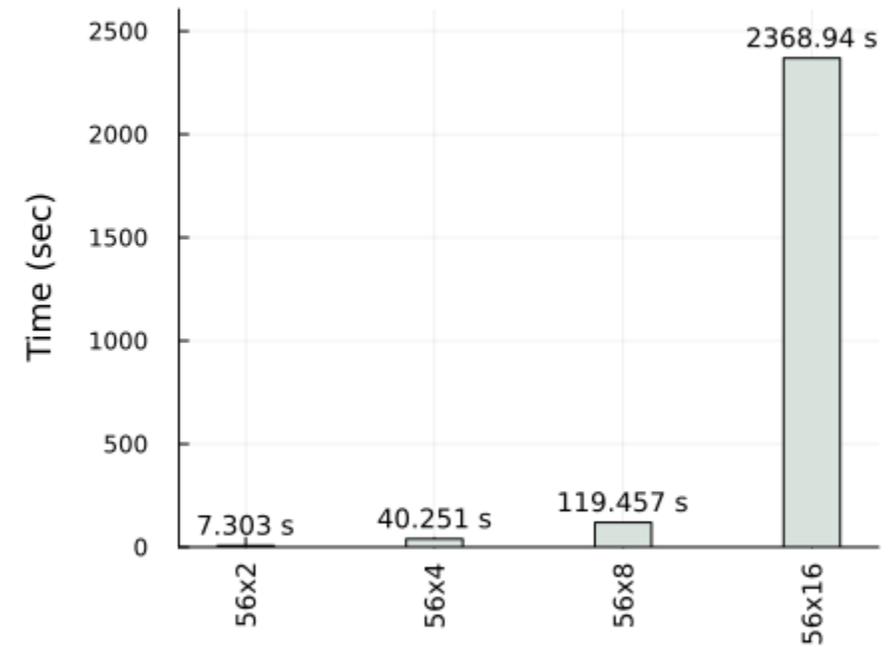


# Model solving time

Model solve time:  
CSAVEinJulia

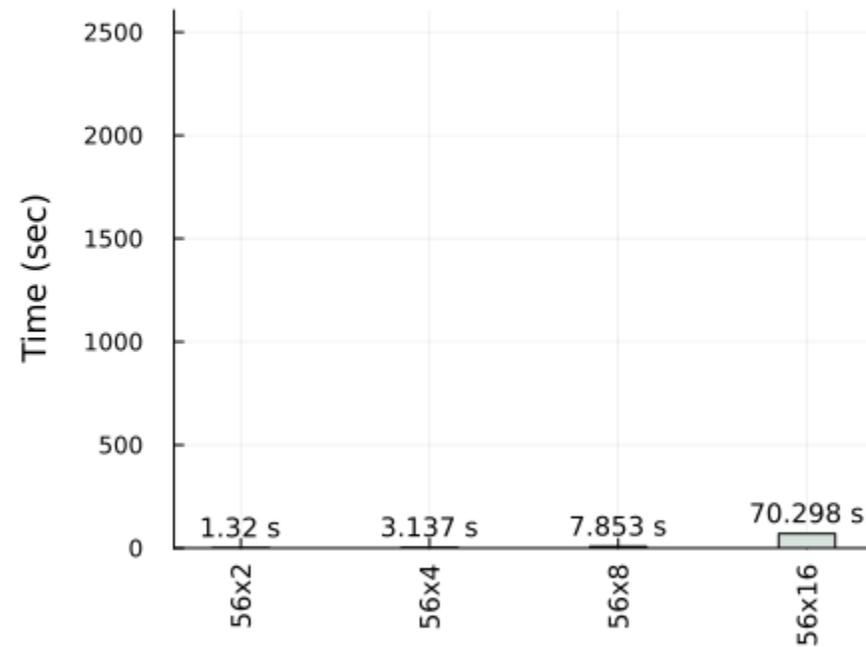


Model solve time:  
CSAVEinGAMS

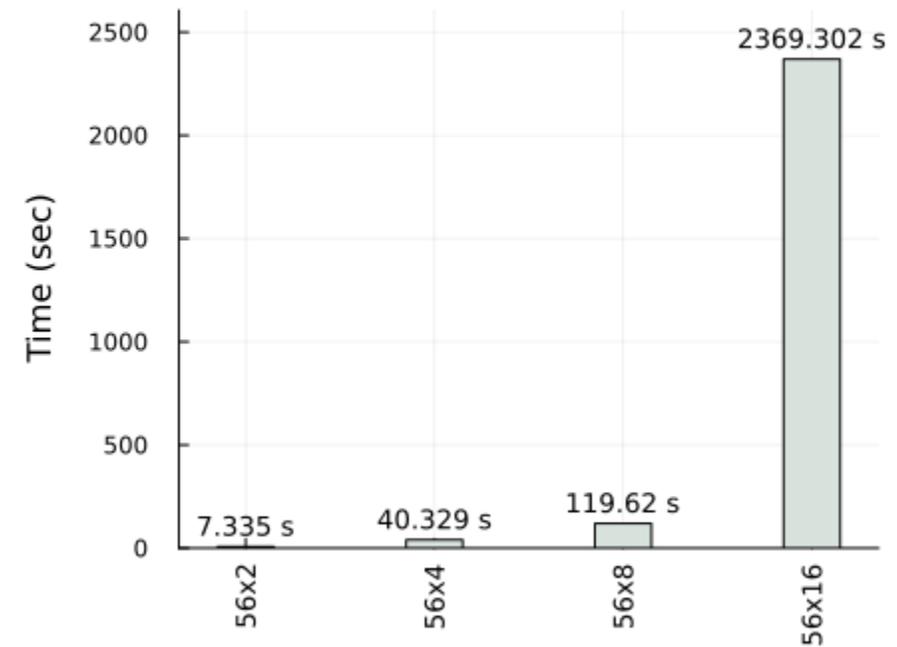


# Model generation + solving time

Model generation + solve time:  
CSAVEinJulia



Model generation + solve time:  
CSAVEinGAMS



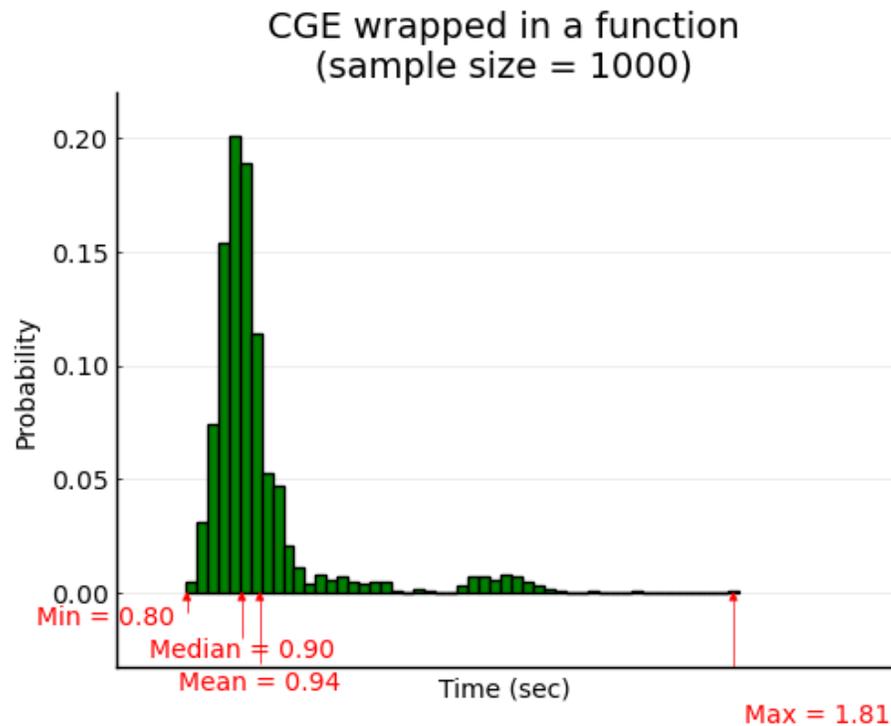
# Looking into Julia solving time

*“Time distributions are always right-skewed... This phenomena can be justified by considering that the machine noise affecting the benchmarking process is, in some sense, inherently positive...”*

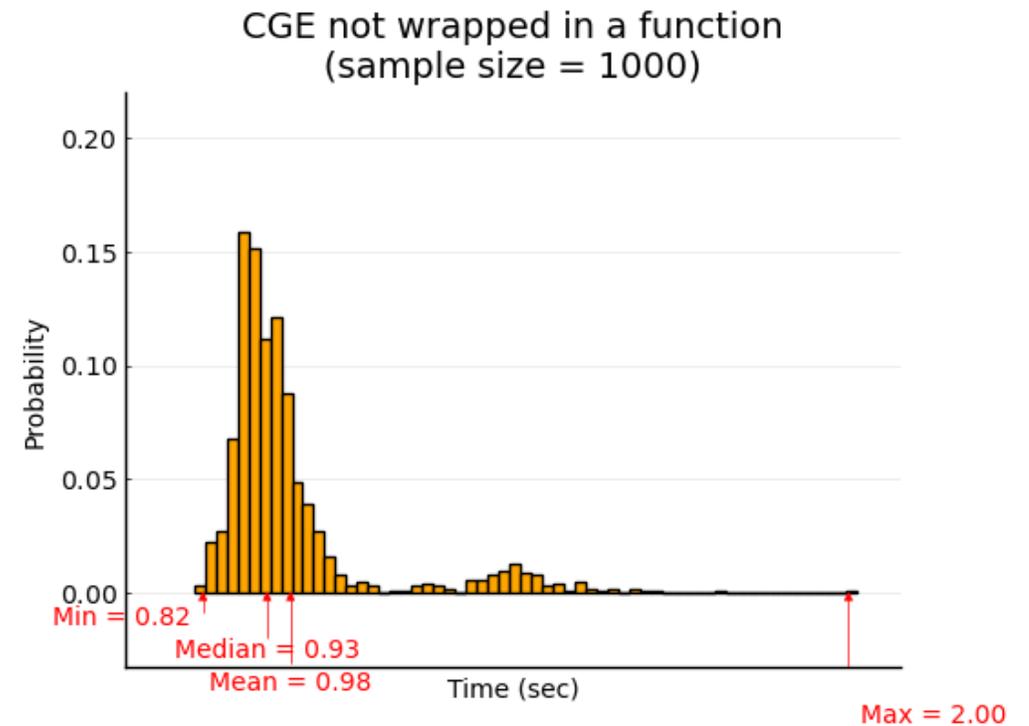
— Chen and Revels (2016)

# Looking into Julia solving time

How solving time is affected by coding strategy?



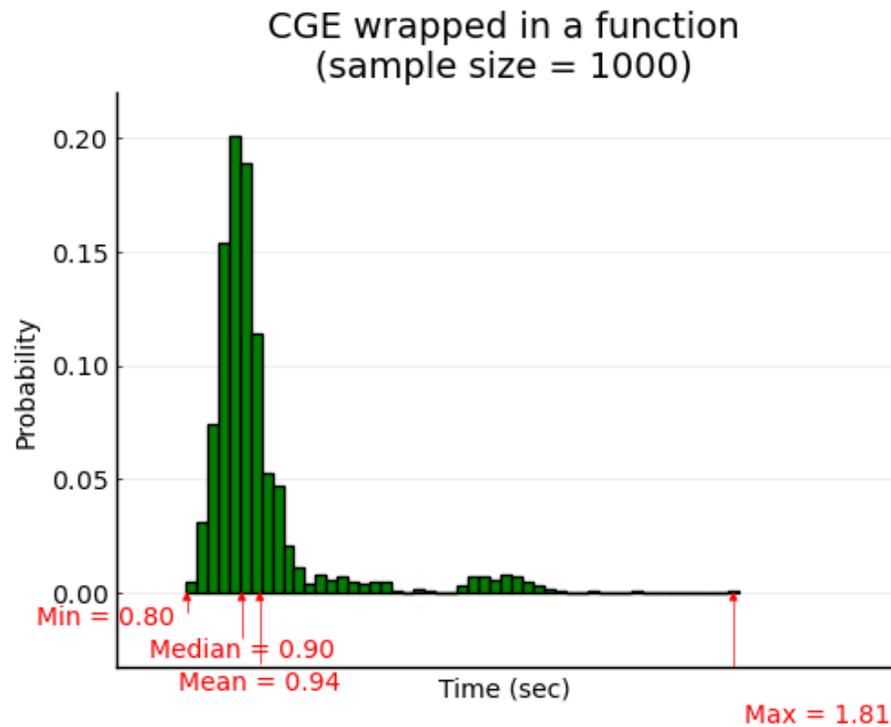
Code lives within the function



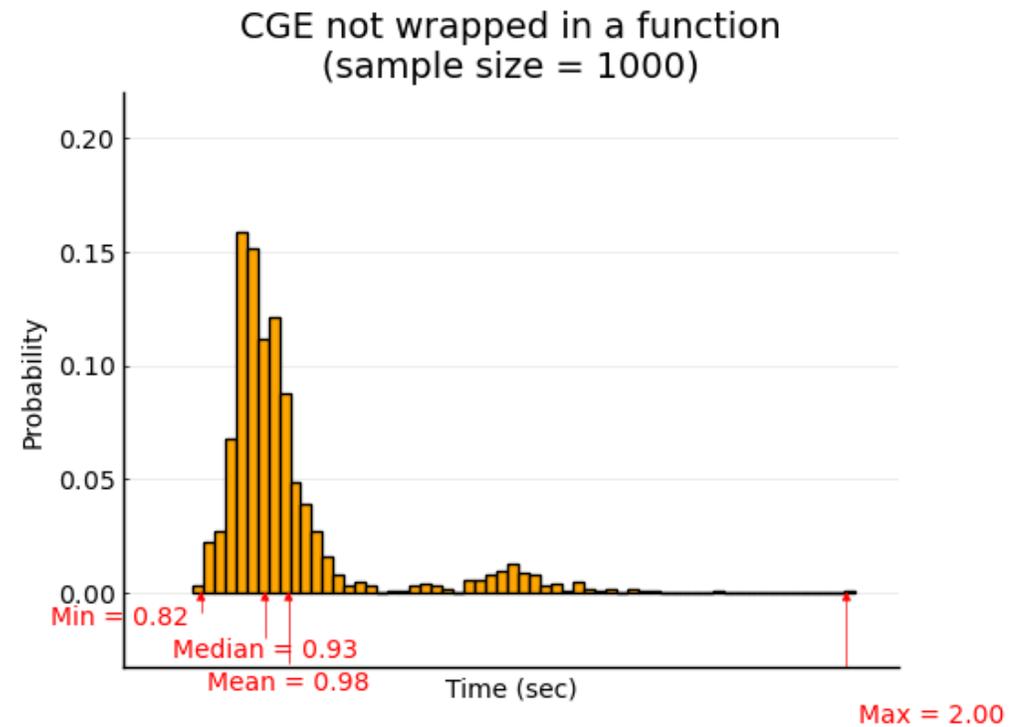
Code lives in global scope

# Looking into Julia solve time

Using median to measure central tendency. Question: Is the difference in medians statistically significant?



Code lives within the function



Code lives in global scope

# Looking into Julia solve time

Bootstrap resampling with sample size  $B = 10000$  (Efron and Tibshirani, 1994)

```
# 3. Bootstrap Loop
for i in 1:B
    # Resample with replacement
    X1_star = sample(X1, n1, replace=true) → 1000-element Vector{Float64}
    X2_star = sample(X2, n2, replace=true)

    # Calculate the statistic (difference in medians)
    delta_star = median(X1_star) - median(X2_star) → Float64

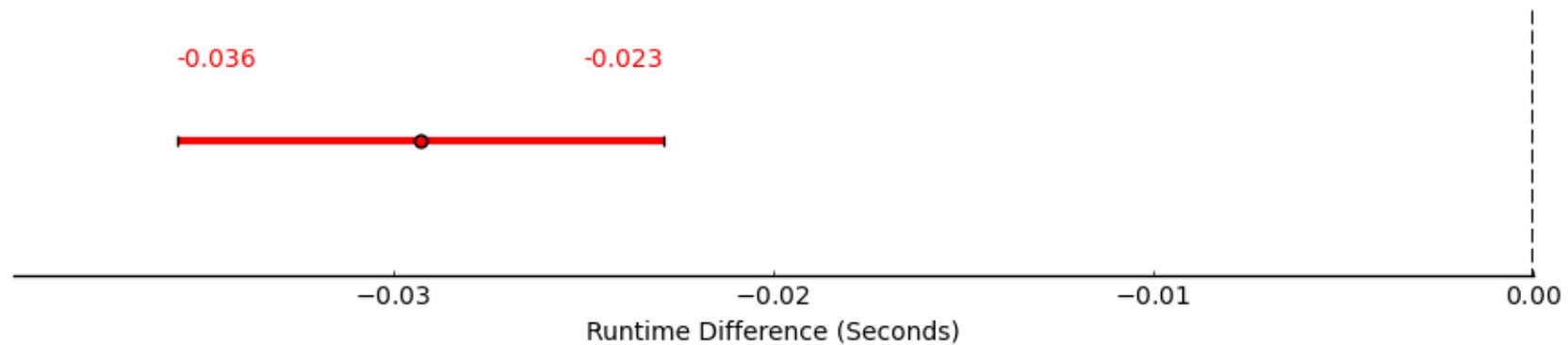
    # Append "delta_star" to the end of an array ("bootstrap_differences") one element at a time.
    push!(bootstrap_differences, delta_star)
end
    After the for-loop is done, bootstrap_differences is a 10000-element Vector{Float64}

# 4. Construct the Confidence Interval (e.g., 95% CI)
CI_lower = quantile(bootstrap_differences, 0.025)
CI_upper = quantile(bootstrap_differences, 0.975)
```

# Looking into Julia solve time

Bootstrap resampling with sample size  $B = 10000$  (Efron and Tibshirani, 1994)

95% Confidence Interval for Median Runtime Difference



# Summary

- Julia

- CSAVEinJulia: slower in model generation
- CSAVEinJulia: faster in solving
- More involved in coding

- GAMS

- CSAVEinGAMS: faster in model generation
- CSAVEinGAMS: slower in solving
- Easier to implement

# Beyond the conclusion

*“A better brush doesn't make a better painter, but a better painter can do more with a better brush.”*

— *Unknown*

# Acknowledgement

*I am thankful for Mitch Phillipson for sharing his knowledge on CGE modeling in Julia, and for Angelo Gurgel who provides me Julia course materials. All errors are my own.*

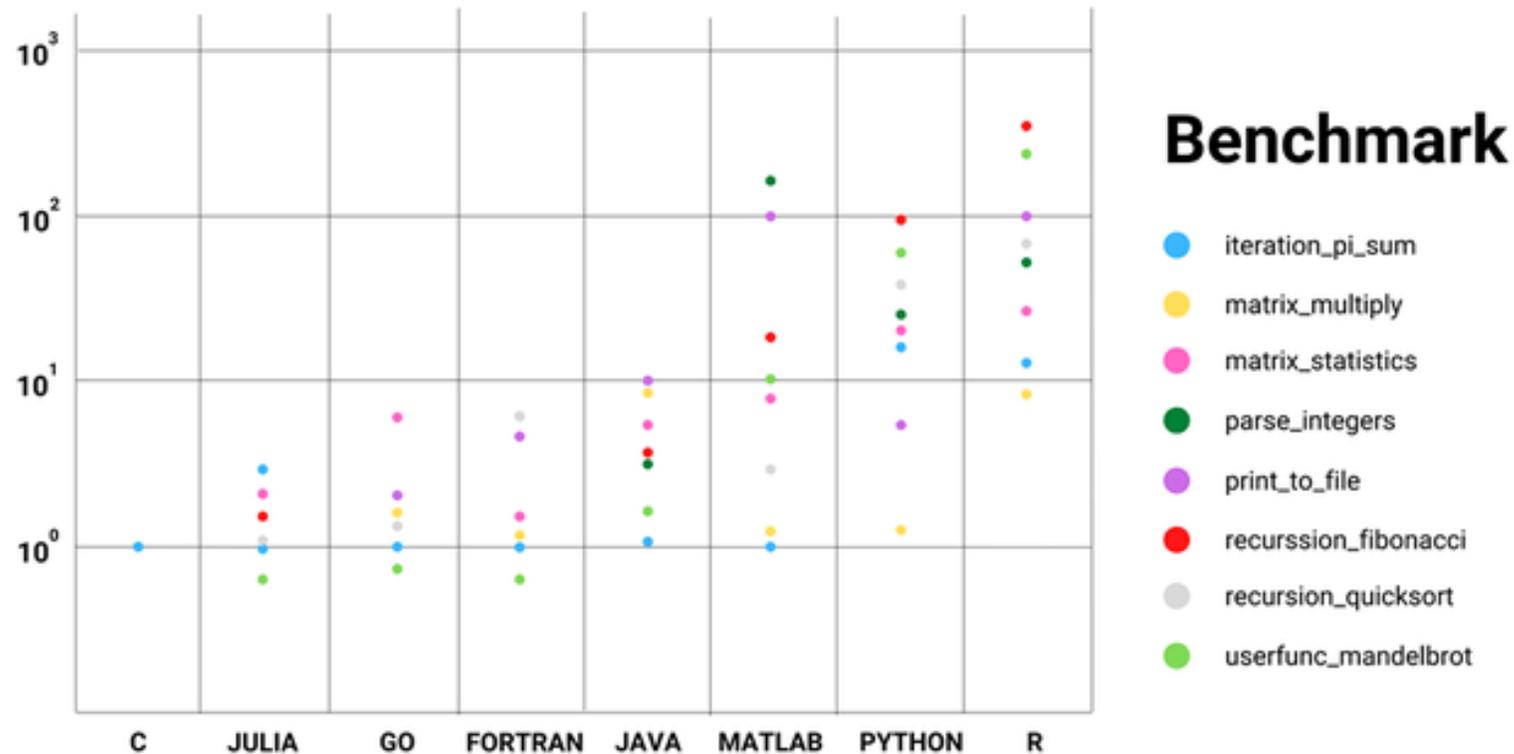
# Thank you!

Questions?

Y.-H. Henry Chen  
chenyh@mit.edu

# Appendix: A comparison

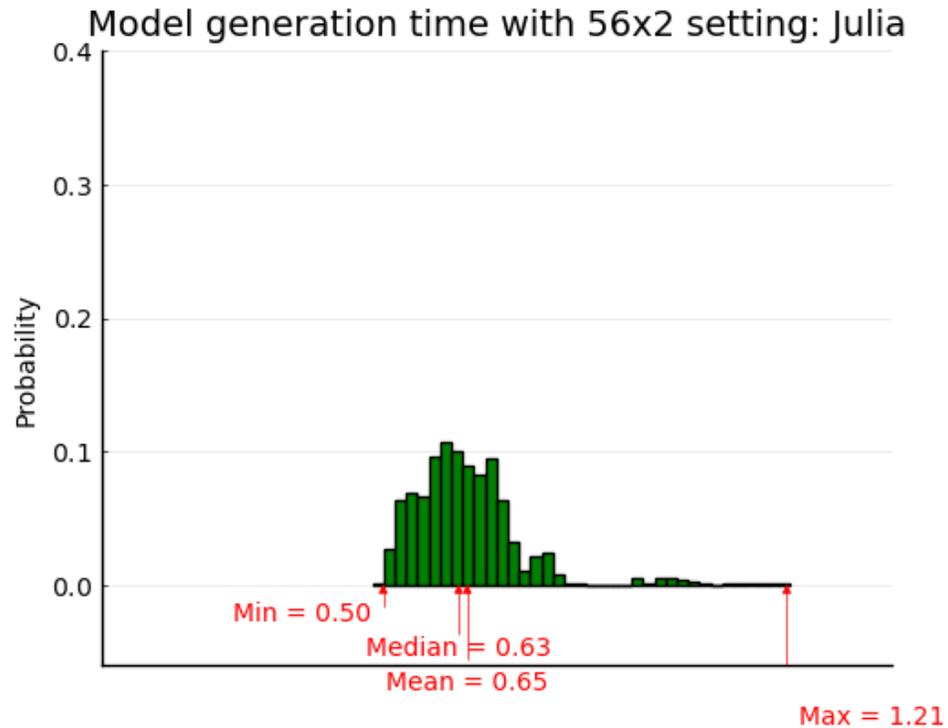
Comparison of Computational Resource for different operations in programming languages



Source: Thakur and Satish (2022)

# Appendix: Model generation time difference

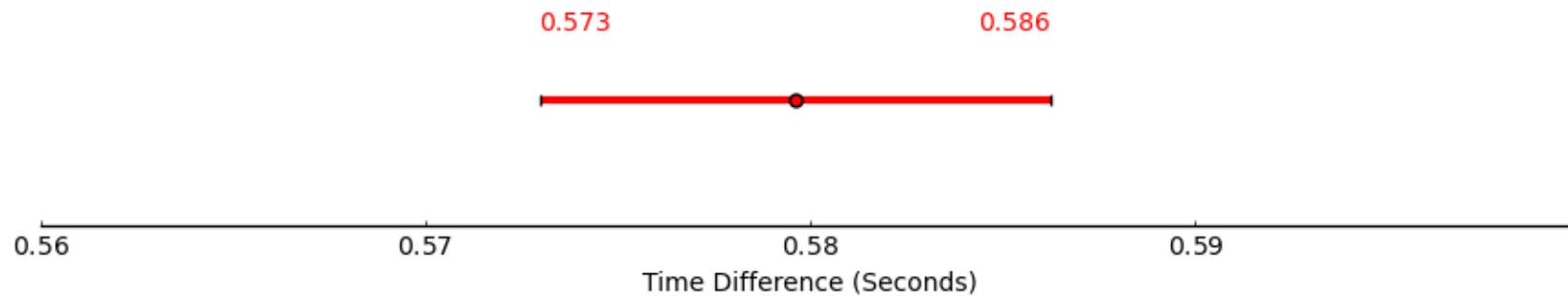
GAMS is faster!



# Appendix: Model generation time difference

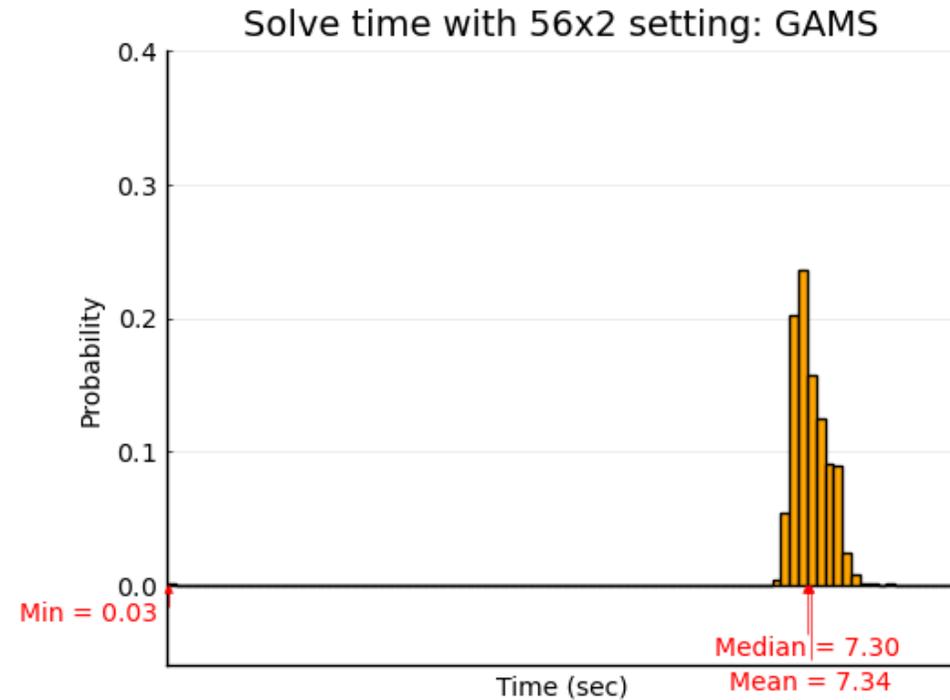
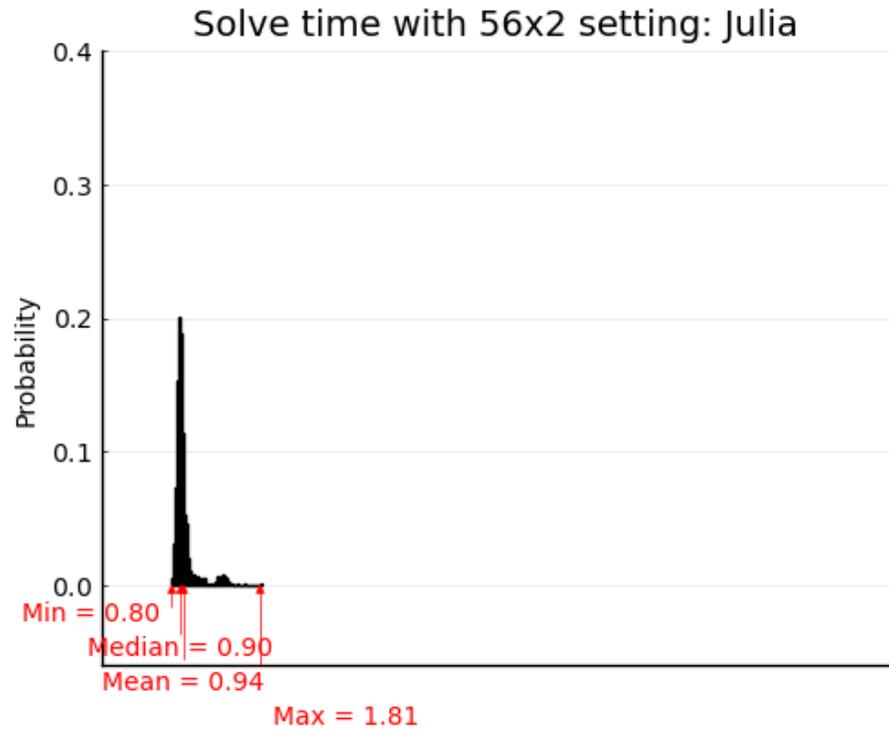
GAMS is faster!

95% Confidence Interval for Median Model Generation Time Difference:  
Julia time minus GAMS time



# Appendix: Solve time difference

Julia is faster!



# Appendix: Solve time difference

Julia is faster!

95% Confidence Interval for Median Solve Time Difference:  
Julia time minus GAMS time

